Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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Wedge-local observables in integrable quantum models with bound states

Karim Shedid Attifa PhD Thesis

Supervisor: Daniela Cadamuro

September 21, 2023

Integrable quantum models (IQMs) 000	Wedge-local observables 0000	IQMs with elementary BS 0000	IQMs with composite BS	Conclusion 000
Motivation: Bound sta	ates in QFT			
 Make up the world (p 	proton/neutron)			
Non-perturbative phe	nomenon / Described by	effective theories	(3) THE 678	MC WINE FOR
Energy of the bound	state below the unbound	states	(3) THE STR FORCE, WHICH	OBEYS, UH
 Correspond to poles i 	n the S-matrix			LL, UMM

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r+1

r + 2

[Weinberg, Quantum Theory of Fields 1]

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r + 1

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r + 2

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... IT HOLDS PROTONS AND NEUTRONS TOGETHER.

Integrable quantum models (IQMs) ●○○	Wedge-local observables 0000	IQMs with elementary BS 0000	IQMs with composite BS 0000	Conclusion 000

Integrable quantum models (IQMs)

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IQMs with elementary BS 0000 IQMs with composite BS 0000 Conclusion 000

Lets look at a simple class of interacting QFTs

Integrable Quantum models:

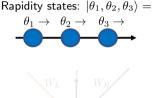
- 1+1 dimensional QFTs
- Interacting
- Fock-space structure z, z^{\dagger}
- Non-perturbatively known S-matrix

Interesting from the perspective of AQFT:

- Operator algebras $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ constructed [Lechner '08]. [Lechner/Alazzawi '17]
- Starting point: Wedge-algebras $\mathcal{A}(W_L + x), \ \mathcal{A}(W_R + y)$

Models (selection) No bound states With bound states

one particle species Massive Ising, Sinh-Gordon Bullough-Dodd





< <p>Image: A matrix

several particles species O(N)-invariant sigma Z(N)-lsing

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IQMs with elementary BS 0000 IQMs with composite BS 0000 Conclusion 000

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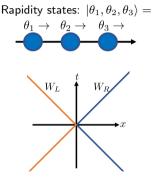
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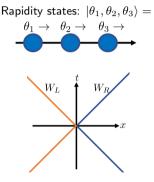
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Models (selection)	one particle species	several particles species
No bound states	Massive Ising, Sinh-Gordon	O(N)-invariant sigma
With bound states	Bullough-Dodd	Z(N)-Ising

Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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The inverse scattering approach

Integrable quantum models are defined by the S-matrix (no Lagrangian needed)

 $S: \mathbb{R} + i[0, \pi] \to \mathbb{C}$ analytic (up to isolated poles)

Hilbert space: S-symmetrized Fock space

$$S(\theta_1 - \theta_2)\Psi_n(\theta_1, \theta_2, ..) = \Psi_n(\theta_2, \theta_1, ..),$$

$$z^{\dagger}(\theta_1)z^{\dagger}(\theta_2) = S(\theta_1 - \theta_2)z^{\dagger}(\theta_2)z^{\dagger}(\theta_1)$$

Bound state structure: poles in the S-matrix

$$p_{\alpha}(\theta + i\theta_{(\alpha\beta)}) + p_{\beta}(\theta - i\theta_{(\beta\alpha)}) = p_{\gamma}(\theta),$$

$$\underset{\theta=0}{\operatorname{Res}} S_{\alpha\beta}\left(\theta + i[\theta_{(\alpha\beta)} + \theta_{(\beta\alpha)}]\right) = R_{\alpha\beta}^{\gamma}$$

$$\left[\operatorname{Note:} p_{\alpha}(\theta) = \begin{pmatrix} m_{\alpha}\cosh\theta\\ m_{\alpha}\sinh\theta \end{pmatrix}\right]$$



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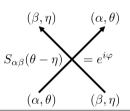
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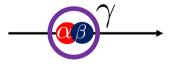
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 $S_{\alpha\beta}(\theta - \eta) = e^{i\varphi}$ (α, θ) (β, η) (β, η)

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Integrable quantum models (IQMs) 000	Wedge-local observables ●○○○	IQMs with elementary BS 0000	IQMs with composite BS	Conclusion 000

Characterization of wedge-local observables

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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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The wedge-local fields				

In IQMs without bound states, the "free field" equivalent is wedge-local: [Lechner '08]

Left-wedge field:
$$\phi(f) = \int d\theta \Big(f^+(\theta) z^{\dagger}(\theta) + f^-(\theta) z(\theta) \Big),$$

 $f^{\pm}(\theta) = \int d^2 x f(x) e^{\pm i p(\theta) \cdot x}$

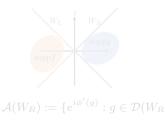
Right-wedge field: $\phi'(g) = J\phi(Jg)J \leftarrow Modular \text{ conjugation},$

Wedge-locality condition:

$$[\phi(f), \phi'(g)] = 0$$
 for $f \in \mathcal{D}(W_L), \ g \in \mathcal{D}(W_R)$

Generators of wedge-algebras

 $\mathcal{A}(W_L) := \{ e^{i\phi(f)} : f \in \mathcal{D}(W_L) \}''$



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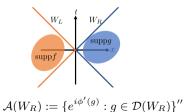
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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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Fock Space structure
$$\rightarrow A = \sum_{m,n \in \mathbb{N}_0} \int \frac{d^m \boldsymbol{\theta} d^n \boldsymbol{\eta}}{m! n!} \int_{\substack{\boldsymbol{\uparrow} \\ \text{connected matrix elements}}} f_{m,n}(\boldsymbol{\theta} | \boldsymbol{\eta}) z^{\dagger m}(\boldsymbol{\theta}) z^n(\boldsymbol{\eta})$$

Matrix elements without Dirac delta terms:

 $\langle z^{\dagger}(\theta_1) z^{\dagger}(\theta_2) \Omega | A z^{\dagger}(\eta) \Omega \rangle =$

$$f_{2,1}(\theta_1, \theta_2 | \eta) + f_{1,0}(\theta_1)\delta(\theta_2 - \eta) + f_{1,0}(\theta_2)S(\theta_1 - \theta_2)\delta(\theta_1 - \eta)$$



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$$\eta$$

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$$\theta_1 + \theta_2$$

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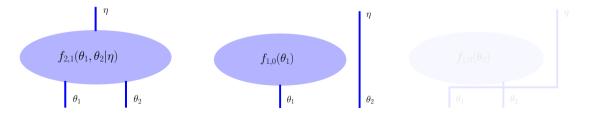
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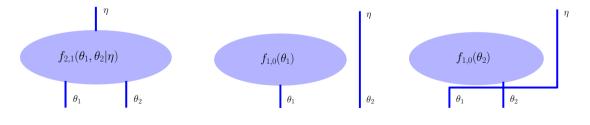
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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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The share and strength				

The characterization theorem

Theorem (Characterization theorem: [Bostelmann/Cadamuro '15])

A wedge-local observable A (in the sense that $[A,\phi'(g)]=0$) satisfies

$$A = \sum_{m,n=0}^{\infty} \int \frac{d^m \boldsymbol{\theta} d^n \boldsymbol{\eta}}{m!n!} F_{m+n}(\boldsymbol{\theta} + i\boldsymbol{0}, \boldsymbol{\eta} + i\boldsymbol{\pi} - i\boldsymbol{0}) z^{\dagger m}(\boldsymbol{\theta}) z^n(\boldsymbol{\eta}).$$

where $F_k : \mathcal{I}^k_+ \to \mathbb{C}$ is analytic and satisfies a list of properties.

- $f_{m,n} \to F_{m+n}$ (many matrix elements are boundary values of the same function).
- F_k highly restricted (analyticity, additional properties)

Examples: (Powers of) the wedge-field.

$$A = \phi(f) | F_1(\theta) = f^+(\theta), \ F_1(\eta + i\pi) = f^-(\eta) \ , \ F_k = 0 \ \text{for} \ k \neq 1,$$

Properties of F_k for local observables are well established (Bootstrap program) [Babujian/Karowski [04].

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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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Wedge-local quantities in IQMs with bound states

 $\phi(f)$ not wedge-local anymore! Has to be modified by a bound state operator: [Cadamuro/Tanimoto '15, '17]

$$\widetilde{\phi}(f) = \phi(f) + \chi(f),$$
$$[\chi(f)\Psi]_{1}^{\gamma}(\theta) = \sum \sqrt{2\pi |R_{\alpha\beta}^{\gamma}|} f_{\alpha}^{+}(\theta + i\theta_{(\alpha\beta)})\Psi_{1}^{\beta}(\theta - i\theta_{(\alpha\beta)})\Psi_{1}^{\beta}($$

Formal expression $\chi(f) = \sum_{\alpha,\beta,\gamma} \int d\theta \ f_{\alpha}^{+}(\theta + i\theta_{(\alpha\beta)}) z_{\gamma}^{\dagger}(\theta) z_{\beta}(\theta - i\theta_{(\beta\alpha)})$



$\widetilde{\phi}(f)$ is not an Araki expansion!

- Quadratic in Zamolodchikov operators.
- Complex shift $\theta i\theta_{(\beta\alpha)}$.

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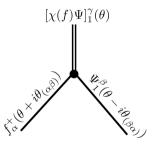
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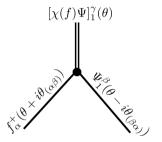
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Characterization theorem in IQMs with bound states [PhD Results]

Theorem (Characterization theorem in scalar IQMs with bound states)

A wedge-local observable A (in the sense that $[A,\phi'(g)+\chi'(g)]=0$) satisfies

$$A = \sum_{m,n=0}^{\infty} \int \frac{d^m \boldsymbol{\theta} d^n \boldsymbol{\eta}}{m! n!} F_{m+n}(\boldsymbol{\theta} + i \boldsymbol{0}, \boldsymbol{\eta} + i \boldsymbol{\pi} - i \boldsymbol{0}) z^{\dagger m}(\boldsymbol{\theta}) z^n(\boldsymbol{\eta}).$$

with F_k as before, except for additional bound state poles

$$2\pi i \operatorname{Res}_{\theta_2 = \theta_1 + i\frac{2\pi}{3}} F_k(\theta_1, \theta_2, \theta_3, \ldots) = -\sqrt{2\pi |R|} F_{k-1}\left(\theta_1 + i\frac{\pi}{3}, \theta_3, \ldots\right)$$

mplications:

- Recursion relation: $F_k \xrightarrow{\text{Res}} F_{k-1} \xrightarrow{\text{Res}} F_{k-2} \cdots$ \Rightarrow Araki expansion of wedge-local observables never terminates (infinite orders)!
- There can be no finite-order wedge-local field of Araki form! (with sufficiently regular F_k)

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Implications:

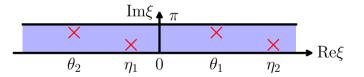
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Sketch of the proof				

$$\langle \Phi_m, [A, \phi'(g)] \Psi_n \rangle \sim \int d\xi \left(g^-(\xi) F_{m+n+1}(\boldsymbol{\theta}, \xi, \boldsymbol{\eta} + i\boldsymbol{\pi}) - g^-(\xi + i\boldsymbol{\pi}) F_{m+n+1}(\boldsymbol{\theta}, \xi + i\boldsymbol{\pi}, \boldsymbol{\eta} + i\boldsymbol{\pi}) \right)$$

Rewrite this as a contour integral

$$\langle \Phi_m, [A, \phi'(g)] \Psi_n \rangle \sim \oint_{\mathbb{R}+i[0,\pi]} d\xi g^-(\xi) F_{m+n+1}(\theta, \xi, \eta + i\pi)$$



 $\langle \Phi_m, [A, \phi'(g) + \chi'(g)] \Psi_n \rangle = 0 \Leftrightarrow$

 $\langle \Phi_m, [A, \phi'(g)] \Psi_n
angle = - \langle \Phi_m, [A, \chi'(g)] \Psi_n
angle \leftarrow \mathsf{Pole} ext{ contributions to the contour integra}$

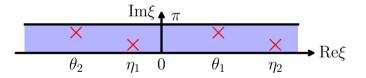
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Sketch of the proof				

$$\langle \Phi_m, [A, \phi'(g)] \Psi_n \rangle \sim \int d\xi \left(g^-(\xi) F_{m+n+1}(\boldsymbol{\theta}, \xi, \boldsymbol{\eta} + i\boldsymbol{\pi}) - g^-(\xi + i\boldsymbol{\pi}) F_{m+n+1}(\boldsymbol{\theta}, \xi + i\boldsymbol{\pi}, \boldsymbol{\eta} + i\boldsymbol{\pi}) \right)$$

Rewrite this as a contour integral

$$\langle \Phi_m, [A, \phi'(g)] \Psi_n \rangle \sim \oint_{\mathbb{R}+i[0,\pi]} d\xi g^-(\xi) F_{m+n+1}(\theta, \xi, \eta + i\pi)$$



 $\langle \Phi_m, [A, \phi'(g) + \chi'(g)] \Psi_n \rangle = 0 \Leftrightarrow$

 $\langle \Phi_m, [A, \phi'(g)] \Psi_n \rangle = - \langle \Phi_m, [A, \chi'(g)] \Psi_n \rangle \leftarrow \text{Pole contributions to the contour integral}$

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Integrable quantum models (IQMs) 000	Wedge-local observables 0000	IQMs with elementary BS 0000	IQMs with composite BS ●000	Conclusion 000
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IQMs with composite bound states

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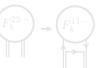
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IQMs with composite BS 0000

Models with "higher-order" bound state structure



In scalar IQMs with bound states, the structure is simple: No composite particles!



$$\operatorname{Res}_{\theta_2=\theta_1+i\frac{\pi}{2}} F_k^{22\cdots}(\theta_1,\theta_2,\ldots) = 2\pi |R| F_k^{1\overline{1}\cdots}\left(\theta_1+i\frac{\pi}{4},\theta_1+i\frac{\pi}{4},\ldots\right)$$

Integrable quantum models (IQMs) 000 Wedge-local observab 0000 IQMs with elementary BS 0000 IQMs with composite BS

Conclusion 000

Models with "higher-order" bound state structure

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The scaling Z(4)-Ising model: Two elementary particles $\{1, \overline{1}\}$ and one composite particle $\{2 = \overline{2}\}$.

Higher-order bound state structure **Double poles** in the S-matrix [Coleman/Thun '74]

 $(F_k^{22\cdots}) \rightarrow (F_k^{11\cdots})$

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Integrable quantum models (IQMs) 000 Wedge-local observab

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 S_{22}

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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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A reversal of strategies				

In models with elementary bound state structure:

Have: Wedge-local field $\phi(f) + \chi(f)$. Want: Proof of form factor properties F_k .

$$[A, \phi'(g) + \chi'(g)] = 0 \Rightarrow F_k$$

In models with composite bound state structure:

Wedge-local field generally unknown $\phi(f) + \chi(f) + X(f)?.$ [Cadamuro/Tanimoto '17]

Idea: Use expected form factor properties as axioms \rightarrow obtain a candidate for the wedge-local field.

Have: Form Factor Axioms (Bootstrap program [Babujian/Foerster/Karowski '06]) Want: Candidate for the wedge-local field by requiring

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QMs with elementary BS

IQMs with composite BS

Conclusion 000

A second-order bound state operator in Z(4)-Ising [PhD Results]

Conjecture (Wedge-local field in the Z(4)-Ising model)

Given the form factor axioms in the Z(4)-Ising model, the wedge-local field (satisfying $[A, \widetilde{\phi}'(g)]$) takes the form

 $\widetilde{\phi}(f) = \phi(f) + \chi(f) + X(f),$

where X(f) is the second-order bound state operator, given by

$$X(f) = 2\pi |R| \int d\theta \left[f_2^+ \left(\theta + i\frac{\pi}{2}\right) z_2^\dagger(\theta) z_1 \left(\theta - i\frac{\pi}{4}\right) z_{\bar{1}} \left(\theta - i\frac{\pi}{4}\right) + h.c. \right]$$

Problems:

- X(f) is not an operator on the Hilbert space (bounded or unbounded), since $[X(f)\Psi](\theta_1, \theta_2) \propto \delta(\theta_1 \theta_2)$.
- [X(f), X'(g)] cannot be computed, because product X(f)X'(g) is ill-defined.
- Limited use in construction of wedge algebras.

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Wedge-local observat

QMs with elementary BS

Conclusion 000

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Integrable quantum models (IQMs) 000	Wedge-local observables 0000	IQMs with elementary BS 0000	IQMs with composite BS 0000	Conclusion ●00
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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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What have we learned?				

Simple bound state structure

Theorem: Characterization of wedge-local observables via form factors

$$A = \sum_{m,n \in \mathbb{N}_0} F_{m+n}(z^{\dagger m} z^n)$$

Insights:

Expansion is always infinite: $F_k \neq 0 \ \forall k \in \mathbb{N}_0$.

Class of expansion distinct from wedge-local field,

$$\widetilde{\phi}(f) = \phi(f) + \chi(f),$$

 $\chi(f) \propto z_{\gamma}^{\dagger}(\theta) z_{\beta}(\theta - i\theta_{(\beta\alpha)})$

(Quadratic in $z^{\dagger}z$, contains complex shift)

Models with composite particles

Conjecture: The wedge-local field has to be supplemented by a "second-order bound state operator",

 $\widetilde{\phi}(f) = \phi(f) + \chi(f) + X(f).$

Insights

More intricate bound state structure \Rightarrow more intricate structure of wedge-local observables.

X(f) ill-defined as an operator \Rightarrow not the right tool for construction of wedge-algebras

$$\mathcal{A}(W_L) := \{ e^{i\widetilde{\phi}(f)} : f \in \mathcal{D}(W_L) \}''$$

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(Quadratic in $z^{\dagger}z$, contains complex shift)

Insights:

More intricate bound state structure \Rightarrow more intricate structure of wedge-local observables.

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Integrable quantum models (IQMs) 000	Wedge-local observables 0000	IQMs with elementary BS 0000	IQMs with composite BS 0000	Conclusion O●O
What have we learned?				
Simple bound state structu	ıre	Models with compos	ite particles	
Theorem: Characterization of via form factors	of wedge-local observables	bles Conjecture: The wedge-local field has to be supplemented by a "second-order bound state opera		operator",
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< <p>Image: A matrix

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Integrable quantum models (IQMs)	Wedge-local observables	IQMs with elementary BS	IQMs with composite BS	Conclusion
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IQMs with elementary B! 0000 IQMs with composite B 0000 Conclusion

Thank you for your attention!

