Quantum General Covariance

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Abstract:

String-localized QFT is an autonomous approach to (perturbative) QFT, not relying on quantization of a given classical field theory. Instead, quantum principles, notably the necessity of a Hilbert space, constrain the form of admissible interactions.

It will be shown that the unique self-interaction of particles of helicity 2 consistent with the said quantum principles coincides with the Einstein(-Hilbert) Lagrangian, and their couplings to matter must coincide with the known interactions whose form is usually credited to general covariance.

General covariance is thus not assumed but derived.

Joint work with Christian Gass and Jose Gracia-Bondia, Class. Qu. Grav. 2023, arXiv:2308.09843

"Common wisdom" says:

Quantum Theory and Einstein-Hilbert Gravity like each other "like cats and dogs".

String-localized quantum field theory says:

Quantum Theory with helicity-2 particles wants Einstein-Hilbert Gravity.

- **QUANTUM FIELDS OF HELICITY 2:**
  - **gauge versus string-localized**
- **COUPLING TO MATTER: THE “LOCK”**
- **SELF-INTERACTIONS: THE “KEY”**
- **EINSTEIN-HILBERT AND GENERAL COVARIANCE**

- **HELICITY TWO QUANTUM FIELDS:**
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Weinberg-Witten quantization of helicity 2

- Wigner unitary representations $U^{\pm 2}$ for helicity 2 = Mackey induced from corresponding unireps $u^{\pm 2}$ of E_2.
- Two creation and innihilation operators $a^{\pm 2}(k)$,$a^{\pm 2*}(k)$. Fock space with **two states per momentum**.
- Weinberg covariant intertwiner construction (at least rank 4)

$$F_{[\mu\kappa][\nu\lambda]}(x) = \int d\mu_0(k) \sum_{\pm} \left[e^{-ikx} a^{\pm 2}(k) u^{\pm 2}_{[\mu\kappa][\nu\lambda]}(k) + h.c.\right].$$

- **Positive definite** two-point function

$$\langle\!\langle F_{[\mu\kappa][\nu\lambda]}(x) F_{[\mu'\kappa'][\nu'\lambda']}(x')\rangle\!\rangle = (F_{[\mu\kappa][\nu\lambda]}(x)\Omega, F_{[\mu'\kappa'][\nu'\lambda']}(x)\Omega)$$

$$= \tfrac{1}{2}\left[\eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\nu\mu'} - \eta_{\mu\nu}\eta_{\mu'\nu'}\right]\partial_\kappa\partial_\lambda\partial'_{\kappa'}\partial'_{\lambda'} W_0(x-x') \begin{array}{l} -[\mu\kappa]-[\mu'\kappa'] \\ -[\nu\lambda]-[\nu'\lambda'] \end{array}$$

- Symmetries of the "linearized Riemann tensor".

Gauge theory approach

- Metric deviation field $h^{\mu\nu}$ in Minkowski background $\sqrt{-g}g^{\mu\nu} =: \eta^{\mu\nu} + \kappa h^{\mu\nu}$.
- Canonical quantization in Feynman gauge:

$$\langle\!\langle h_{\mu\nu}(x)h_{\mu'\nu'}(x')\rangle\!\rangle = \tfrac{1}{2}\left[\eta_{\mu\mu'}\eta_{\nu\nu'}+\eta_{\mu\nu'}\eta_{\nu\mu'}-\eta_{\mu\nu}\eta_{\mu'\nu'}\right]W_0(x-x')$$

- 10 components (not conserved, not traceless), 10 states per momentum, indefinite two-point function
- Impose subsidiary conditions, ghosts, BRST $\Rightarrow$ Hilbert space $\mathcal{H}$.
- Only the linearized Riemann tensor $R_{[\mu\kappa][\nu\lambda]} = \frac{\kappa}{2}F_{[\mu\kappa][\nu\lambda]}$ is defined on $\mathcal{H}$:

$$F_{[\mu\kappa][\nu\lambda]} = \partial_\mu\partial_\nu h_{\kappa\lambda} - \partial_\kappa\partial_\nu h_{\mu\lambda} - \partial_\mu\partial_\lambda h_{\kappa\nu} + \partial_\kappa\partial_\lambda h_{\mu\nu}.$$

- Unitarily equivalent to $F_{[\mu\kappa][\nu\lambda]}$ on Wigner Fock space.

- Starts with indefinite state space.
- Return to Hilbert space:

$$10\,\text{d.o.f.} + \text{ghosts} - \text{BRST} = 2\,\text{d.o.f.}$$

- $h_{\mu\nu}$ does not exist on the Hilbert space.

Should this really be the "method of choice"?

String-localized free field

- Start with $F_{[\mu\kappa][\nu\lambda]}$ on Wigner Hilbert space.
- Define
  $$h_{\mu\nu}(x,e_1,e_2) := \int_0^\infty ds_1 \int_0^\infty ds_2\, F_{[\mu\kappa][\nu\lambda]}(x+s_1e_1+s_2e_2)e^\kappa e^\lambda.$$
  Here, e_i are two spacelike vectors, so that $h_{\mu\nu}(x,e_1,e_2)$ is localized in the region $x+\mathbb{R}_+e_1+\mathbb{R}_+e_2$.
- Gives back the local field
  $$\partial_\mu\partial_\nu h_{\kappa\lambda}(e_i)-\partial_\kappa\partial_\nu h_{\mu\lambda}(e_i)-\partial_\mu\partial_\lambda h_{\kappa\nu}(e_i)+\partial_\kappa\partial_\lambda h_{\mu\nu}(e_i)=F_{[\mu\kappa][\nu\lambda]}.$$
- Two degrees of freedom by construction:
  $$\partial^\mu h_{\mu\nu}(x,e_1,e_2)=0, \qquad \eta^{\mu\nu}h_{\mu\nu}(x,e_1,e_2)=0.$$

- Needs smearing in e_i with smooth $c(e_i)$:
$$h_{\mu\nu}(x,c) = I_c^\kappa I_c^\lambda \big(F_{[\mu\kappa][\nu\lambda]}\big)(x).$$
The notation "I_c^κ" includes the integration along $x + \mathbb{R}_+ e$, the contraction with e^κ, and the smearing with $c(e)$.
- The previous properties remain true provided $\int de\, c(e) = 1$ ("unit weight").
- Depends on arbitrary choice of $c(e)$ of unit weight.
- **The string-variation is a derivative:**
$$\delta_c(h_{\mu\nu}(x,c)) = \partial_\mu w_\nu(x,c) + \partial_\nu w_\mu(x,c)$$
with another string-localized field w_μ.
- δ_c "looks like an operator-valued gauge transformation".

- Positive two-point function

$$\langle\!\langle h_{\mu\nu}(x,c) h_{\mu'\nu'}(x',c)\rangle\!\rangle = \tfrac{1}{2}\big[E'_{\mu\mu'}E'_{\nu\nu'} + E'_{\mu\nu'}E'_{\nu\mu'} - E_{\mu\nu}E''_{\mu'\nu'}\big]W_0(x-x')$$

with $E, E', E'' = \eta +$ derivatives of string-integration operators acting on W_0

- Kinematic propagator violates trace condition $\eta^{\mu\nu}h_{\mu\nu}(c) = 0$:

$$\langle\!\langle T_{\rm kin} h_{\mu\nu} h'_{\mu'\nu'}\rangle\!\rangle := \tfrac{1}{2}\big[E'_{\mu\mu'}E'_{\nu\nu'} + E'_{\mu\nu'}E'_{\nu\mu'} - E_{\mu\nu}E''_{\mu'\nu'}\big]T_0(x-x').$$

- Renormalize to make it traceless:

$$\langle\!\langle T_{\rm ren} h_{\mu\nu} h'_{\mu'\nu'}\rangle\!\rangle = \langle\!\langle T_{\rm kin} h_{\mu\nu} h'_{\mu'\nu'}\rangle\!\rangle + T_r(x-x') + \sum_{i=1}^{6} c_i T_{r,i}(x-x')$$

with T_r and $T_{i,r}$ involving string-integrated δ-functions.

The only redundancy is the choice of the smearing function $c(e)$ of unit weight. (May have arbitrarily narrow support.)

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Coupling to matter

- $L_{\mathrm{mat},1}(x,c) = \frac{1}{2} h_{\mu\nu}(x,c)\Theta^{\mu\nu}_{\mathrm{mat}}(x)$.
- Standard stress-energy tensors of matter. Violate Ward identity:

$$\partial^\mu \langle\!\langle T\Theta^{\mu\nu}_{\mathrm{mat}}(x)\Theta^{\rho\sigma}_{\mathrm{mat}}(x')\rangle\!\rangle \neq 0$$

  (some messy explicit expressions, depending on the type (scalar, Dirac, Maxwell) of matter.)
- Perturbative S-matrix at tree level

$$S_{\mathrm{mat}} = Te^{i\kappa \int dx\, L_{\mathrm{mat},1}(x,c)} = \mathbf{1} + i\kappa S^{(1)} + \frac{(i\kappa^2)}{2} S^{(2)} + \dots$$

Tree level is sufficient to determine the interactions!

- The S-matrix must be independent of the meaningless choice of $c(e)$ (**"String Independence" SI**).
- The first-order SI is trivial because $\delta_c(L_{\mathrm{mat},1})$ is a derivative by conservation of Θ_{mat}:

$$\delta_c(S^{(1)}) = \int dx\, \delta_c(L_{\mathrm{mat},1}(x,c)) = \int \partial_\mu w_\nu\, \Theta^{\mu\nu}_{\mathrm{mat}} = \int \partial_\mu\big(w_\nu\, \Theta^{\mu\nu}_{\mathrm{mat}}\big) = 0.$$

- Second-order SI

$$\delta_c(S^{(2)}) = \iint \delta_c\big(T(h\Theta_{\mathrm{mat}})(h'\Theta'_{\mathrm{mat}})\big)\big|^{\mathrm{tree}} \stackrel{!}{=} 0.$$

- The contribution from $h\,h' = \langle\!\langle Thh' \rangle\!\rangle = \eta +$ derivatives vanishes by conservation of Θ_{mat}.

- The contribution
$$\tfrac{1}{4}\delta_c(hh')\Theta_{\mathrm{mat}}\,\Theta'_{\mathrm{mat}} = (\partial w\, h' + h\,\partial w')\cdot T\Theta_{\mathrm{mat}}\Theta'_{\mathrm{mat}}$$
does not vanish because of violation of Ward identities.
- Explicit computation for all three types of matter:
$$\tfrac{1}{4}\delta_c(hh')\Theta_{\mathrm{mat}}\,\Theta'_{\mathrm{mat}} = \text{derivatives} + i\delta_c(L_{\mathrm{mat},2})\delta(x-x') + O_{\mathrm{mat},2}(x,x')$$
with the **"universal obstruction"**
$$O_{\mathrm{mat},2}(x,x') = -i\Theta^{\mu\nu}_{\mathrm{mat}}w^\kappa\big(\partial_\mu h_{\kappa\nu} + \partial_\nu h_{\kappa\mu} - \partial_\kappa h_{\mu\nu}\big)\delta(x-x'),$$
- The derivatives do not contribute to $\delta_c(S)$. The second term can cancelled by adding the **"induced second-order interaction"** $\frac{\kappa^2}{2}L_{\mathrm{mat},2}$ to L_{mat}. (More on $L_{\mathrm{mat},2}$ later.)
- The obstruction term cannot be cancelled. The second-order SI cannot be fulfilled. The perturbation theory is inconsistent, because it depends on the meaningless quantity $c(e)$.
- This is what we call **"the lock"**.

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The key to open the lock is the helicity-2 self-interaction:

- In first order, $L_{\mathrm{self}} = \kappa L_{1,\mathrm{self}} + \dots$ must satisfy

$$\delta_c(L_{1,\mathrm{self}}) = \text{derivatives},$$

  in order that the S-matrix

$$S = Te^{i\int(L_{\mathrm{self}}+L_{\mathrm{mat}})}$$

  still fulfills SI in first order. This fixes (up to derivatives and an overall factor) the unique cubic self-interaction

$$L_{\mathrm{self},1}(c) = \text{three terms involving of structure } h\cdot\partial h cdot \partial h.$$

- The computation in second order yields

$$L_{\mathrm{self},1}\, L'_{\mathrm{self},1} = \text{derivatives} + i\delta_c(L_{\mathrm{self},2})\delta(x-x').$$

  with $L_{\mathrm{self},2} =$ three terms $h\cdot h\cdot\partial h\cdot\partial h$. The derivatives do not contribute to $\delta_c(S)$. The second term can cancelled by adding the **"induced second-order self-interaction"** $\frac{\kappa^2}{2}L_{\mathrm{self},2}$ to L_{self}.
- Self-interactions of helicity-2 particles are separably consistent to second order.

- It remains to consider the mixed terms

$$\delta_c\big(\underbrace{L_{\mathrm{self},1}\, L'_{\mathrm{mat},1}} + \underbrace{L_{\mathrm{mat},1}\, L'_{\mathrm{self},1}}\big) = \tfrac{1}{2}\big(\underbrace{L_{\mathrm{self},1}\, h'_{\mu\nu}}\, \Theta'^{\mu\nu}_{\mathrm{mat}} + \Theta^{\mu\nu}_{\mathrm{mat}}\, \underbrace{h_{\mu\nu}\, L'_{\mathrm{self},1}}\big).$$

- Obviously contains a factor Θ_{mat}. The computation yields

$$= \text{derivatives} + i\Theta^{\mu\nu}_{\mathrm{mat}} w^\kappa \big(\partial_\mu h_{\kappa\nu} + \partial_\nu h_{\kappa\mu} - \partial_\kappa h_{\mu\nu}\big)\delta(x - x'),$$

  which exactly cancels the previous $O_{\mathrm{mat},2}(x, x')$, provided the overall factor of $L_{\mathrm{self},1}$ was correctly adjusted.
- **The key has opened the lock.**
- To second order, the interaction is

$$L = \kappa L_{\mathrm{self},1} + \frac{\kappa^2}{2} L_{\mathrm{self},2} + \kappa L_{\mathrm{mat},1} + \frac{\kappa^2}{2} L_{\mathrm{mat},2} + \dots$$

- The induced interactions $L_{\mathrm{self},1}$ and $L_{\mathrm{mat},1}$ (for each matter type) are uniquely determined by the need to secure SI, i.e., the need to get an S-matrix that does not depend on the meaningless $c(e)$.

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- Recall the definition $\sqrt{-g}g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}$ of the classical metric deviation. $\kappa^2 = 32\pi G_N$ is related to Newton's constant.
- Expand the Einstein-Hilbert action (dictated by general covariance)
$$\frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g(x)}R(x)$$
in terms of h, beginning with quadratic terms $\partial h \cdot \partial h$ and $h \cdot \partial\partial h$.
- One may remove a total derivative (irrelevant for the classical equations of motion) to eliminate all second derivatives. One arrives at the Einstein action.
- The quadratic terms determine the "canonical quantization".
- When gauge conditions are imposed (Hilbert gauge: $\partial_\mu h^{\mu\nu} = 0$, trace $\eta^{\mu\nu}h_{\mu\nu} = 0$), the cubic and quartic terms are of structure $h \cdot \partial h \cdot \partial h$ and $h \cdot h \cdot \partial h \cdot \partial h$ (no second derivatives).

SI, that is, quantum consistency of string-localized helicity-2 interactions imply general covariance:

- The self-interactions $L_{\text{self},1}(c)$ and $L_{\text{self},2}(c)$, as determined by SI in first and second order, **coincide** with the cubic and quartic terms of the Einstein(-Hilbert) Lagrangian, in which the classical $h_{\mu\nu}$ is replaced by the free string-localized quantum field $h_{\mu\nu}(c)$ (with normal ordering).
  Independent of the propagator renormalization constants c_i.
- Recall that $h_{\mu\nu}(c)$ by construction satisfies the "gauge conditions" that have to be imposed "by hand" on $h_{\mu\nu}$.
- The matter interactions $L_{\text{mat},1}$ and $L_{\text{mat},2}$ **coincide** with the cubic and quartic terms ($=$ linear and quadratic in h) of the generally covariant free matter Lagrangians, upon the same substitution, and case by case for scalar, Dirac and Maxwell matter.

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- **CONCLUSION AND PERSPECTIVE**

- Higher order not yet done (compare analogous work by Dütsch in the setting of "perturbative gauge invariance").
- The tree level recursively fixes the interactions. Loop renormalization to be done! (Result by Gaß provides essential preriquisites for Epstein-Glaser renormalization.)
- No power counting renormalizability. Speculation: SI could turn out to be so powerful as to fix renormalization constants beyond power counting. (??)
- String-independent interacting local observables?
  - Compute interacting fields in two steps: the intermediate step decides whether an interacting field will be local or string-localized.
  - The former will be "local observables" (in the present fixed-background setting, we expect $F_{[\mu\kappa][\nu\lambda]}|_{L(c)}$ to be local). The latter live on a (possibly superselection-extended) Hilbert space and may interpolate between sectors (Example QED).

- Similar "lock-key" scenarios exist also in **SM physics**:
  - "Minimal" interactions $A^a_\mu(c) j^\mu_a$ of massless gluons with fermions satisfy SI at first order, but are inconsistent at second order. Adding the cubic gluon self-interaction resolves the obstruction, and determines the quartic self-interaction.
  - Massive vector bosons (W, Z) can be coupled to chiral fermions (without a Higgs mechanism to make them massive). Their non-abelian self-interaction is inconsistent, unless one adds a coupling to a scalar field ("Higgs") with a Higgs potential.
- The string-localized interacting fields of QED include the Dirac field. Its non-locality is necessary to be compatible with the global Gauß Law.
- SQFT reproduces gauge theories without assuming gauge or diffeomorphism invariance.